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## TEACHING PUPILS HOW TO STUDY MATHEMATICS

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### *Continued*

An investigation with the high school pupils of St. Louis indicates that there is probably not so much difficulty with the study of algebra as there is with geometry. At least the pupils say they enjoy algebra more. There may not be the need for a detailed discussion of the teaching and study of algebra that there was in the case of geometry. However, algebra should be constantly related to the study of arithmetic. This will make many of its difficulties clear to the pupil who finds trouble in it. We spend too much time on the formal processes as such. Just as it was discovered that the best way to learn the alphabet was incidentally through reading, and not as a thing of importance in itself; so, much of the formal work in algebra is best learned through the thought or translation problems. The writer has heard teachers contend that elementary algebra should be confined almost wholly to the mastery of, and drill in, the formal processes. These processes must be mastered, but much of the motive for work, and the maintenance of interest, can best be achieved through its use in real and applied problems—problems which have a vital interest for the pupil although they may have no great value in adult life. Of course no one will question that the ultimate outcome of an elementary course ought to be a thorough knowledge and mastery of the principles of the subject along with skill in their application; and that these results ought to be achieved in the shortest and most attractive way possible. The subject, however, should not be taught by the novice; who, for fear he could not teach anything else, is given a class in algebra.

Much depends upon the attitude of the student towards his work. He must be willing to pay the price of success; he must make the effort necessary for the mastery of mathematics. Such is the price of success in any line of effort. The mastery of mathematics points out his deficiencies, and points the best road to success. Ability to study mathematics means, as a rule, ability to be a good student; and a good student has, many

times over, the chance to succeed in life that a poor student has. Professor Schultze, in "Teaching of Mathematics in Secondary Schools," says:

"It is a common experience to see a pupil in the upper grades suddenly wake up to the meaning of mathematics and thereby change his attitude towards study in general."

Professor Rietz has shown that the Dartmouth students who excelled in mathematics thereby increased their chances of success in law.

A study by President Lowell, of Harvard University, published in the *Educational Review*, October, 1911, indicates similar results for the Harvard students. Of 609 who graduated from college with A.B. plain, only 6.6 per cent. obtained *cum laudi* in the law school. Students of mathematics attained highest honors in the law school. Students of the classics stood next. The qualities of diligence and perseverance, and intensity of application acquired in the study of mathematics secured a higher degree of success than was attained by those who did not study mathematics. If, then, the student finds mathematics hard, let him profit by the determination of Robert Bruce, who, after watching the spider, after many futile efforts finally reach the ceiling, went out and won in battle against odds. If he is inclined to waste time let him get inspiration from the words of Hotspur before the battle of Shrewsbury:

"Oh, gentlemen, the time of life is short;  
To spend that shortness basely were too long,  
If life did ride upon a dial's point,  
Still ending at the arrival on an hour."

Many of the failures in mathematics are due to indifference and laziness on the part of students. Professor Helen A. Merrill, in *MATHEMATICS TEACHER* for December, 1918, shows that obstinacy and neglect on the part of students is responsible for some of the failures in mathematics. Students assume the subject hard and refuse to make effort in it. Or it frequently happens that neglect at some point of the course has destroyed the sequence, and the pupil is not willing to make the effort to make up the loss. This is a subject which demands, almost more than any other, constant diligence and effort, alertness on the

part of the pupil that nothing passes by without being thoroughly understood. This is one of the great values of the subject. But the pupil fails to realize the seriousness of the situation; he refuses to make effort. The words of the foreigner who had fallen into the water and called, "I will drown, nobody shall help me," apply to him.

An interesting study by President Foster, of Reed College, the results of which he published in *Harper's Monthly* for September, 1916, gives an answer to the question, "Are good students in high school more likely than others to become good students in college?" Three colleges in as many states were considered. Of hundreds of students in the University of Wisconsin, above 80 per cent. of those in the first quarter in the high school remained in the upper half of their classes throughout the four years of their university course, and above 80 per cent. of those who were in the lowest quarter in the high school did not rise above mediocre scholarship in the university. Only one in five hundred of those in the lowest quarter reached highest rank in the university. The University of Chicago found that students who failed to receive in high school an average higher than "passing" by at least one-fourth of the difference between the passing mark and 100 per cent, failed in college; such students are therefore, not admitted; where exceptions are made the record in college is seldom satisfactory. Reed College at its fall opening nine years ago admitted only those students who ranked in the first third in the preparatory schools: about 20 per cent. were exceptions to this rule and 2 per cent. were below the median line; these exceptions were selected as the best below the first third. Of these exceptions, practically none rose above the lowest quarter in their college classes. The same results are shown to be true of those who go from college to the professional schools. Surely "promise in high school becomes performance in college," and the mediocre in high school are practically out of the race. President Foster says:

"If all these studies prove anything, they prove that there is a long chain of causal connections binding together the achievements of a man's life and explaining the success of a given moment. . . . Luck is about as likely to strike a man as lightning and about as likely to do him any good. The best

luck a young man can have is the firm conviction that there is no such thing as luck and that he will gain in life just about what he deserves and no more. . . . Nothing seems to promise failure in the tasks of tomorrow with greater certainty than failure in the studies of today. . . . Among teachers the greatest number of criminals are not those who kill their young charges with overwork, but those who allow them to form the habit of being satisfied with less than the very best there is in them."

Much of the inefficiency of students in mathematics in these days is due to the idea somewhat prevalent in the elementary schools, and even found in the high schools, that a student must gain his education by being amused. He is not trained to feel or to know personal responsibility in the matter. His education is entirely a problem for the teacher, and others, to solve. The pupil even resents any effort to secure his initiative if this would suggest anything like drudgery or hard thinking on his part. If it is true that there can be no real education without interest, it must also be remembered that the pupil has some responsibility in the matter of his own interest. Education involves effort, is in direct proportion to effort, and interest is the product of effort. Any pupil with proper instruction can master the mathematics required of him in our high schools, and mastery will give interest and pleasure.

Regarding the easy drifting which pupils are allowed to do in many of our schools today, Dr. Munsterberg is quoted as saying in the *Metropolitan Magazine*, 1910:

"The community has found out that such schemes may be well fitted to give the children a good time at school but lead them to a bad time afterward. Life is hard work and if they have never learned in school to give their concentrated attention to that which does not appeal to them and which does not interest them immediately, they have missed the most valuable lesson of their school years. It has always been found that it is the general education that pays best and the more the period of cultural work can be expanded the more efficient will be the services of the school for the practical services of the nation."

Professor Bagley, of Teachers' College, says in *Teachers' College Record* for November, 1918:

"There has, indeed, been a tendency in both educational theory and school practice to belittle duty and obligation as motives to effort. . . . There is a tendency, indeed, to go even further than this, and to insist that the learner should not be expected or encouraged—much less required—to put forth effort unless he can see clearly whither the effort leads and why the goal is desirable.

"This extreme view would have a tendency, obviously, to encourage a most unfortunate type of individualism. . . . Under the dominance of this extreme reaction against duty and effort, indeed, good schools have degenerated, and their pupils have suffered a real injustice. . . . To impress upon even young children the notion of education as a democratic duty is not beyond the ability of the skillful teacher. . . . The day of individualism has gone by, and the notion of education as a means of securing an individual advantage in the social order should be and can be replaced by the conception of education as the democratic duty. . . . One need have no fear that the emphasis on education as an obligation of the individual to democracy will encourage the barren formalism which all thoughtful students of education have lamented and striven to remedy."

Another source of trouble is the assumption that when a pupil masters a principle this is a guarantee of his ability to apply it. Such is not a necessary result. Many of our best students appear sadly inefficient when they leave high school because they have not been taught the use of their acquisitions. Application is quite as important a part of our work as the teaching of principles. Of course no pupil can be taught all possible applications of his knowledge, but he can be taught *how* to apply it. Professor C. H. Judd, in "The Psychology of High School Subjects," says:

"It is contrary to experience to assume that students can apply mathematics to the other sciences or to the practical affairs of life unless they are trained to see mathematical relations in other forms than those in which they are commonly presented in the schools. The student who knows the abstract demonstra-

tions of geometry, but does not realize that knowledge of space is involved in every manufacturing operation, in every adjustment of agriculture and practical mechanics, is only half trained. Application must be a phase, and an explicit phase, of school work. Application is as different from pure science as pure sciences are different from each other."

The study of mathematics in our high schools could be greatly facilitated by a more skillful classification of pupils. All normal pupils have ability to study mathematics if the subject is properly taught. Those who cannot study the subject are probably no more numerous than physical defectives, and in many cases may belong in special institutions. Every one needs to study mathematics to be ordinarily intelligent on many topics of common interest; yet there are, without doubt, pupils who are not as apt as others in the study of the subject. Pupils can probably be divided into three fairly distinct classes, for purposes of instruction; those of more than ordinary ability; those of medium ability—the largest class; and those below normal. At the present time we are obliged to take pupils of all grades of intelligence and teach them the same things, in the same way, in the hopes of getting the same results from all. This is, however, impossible. Mr. Gruenberg, in *School and Society* for February 15, 1919, says:

"It is desirable that teachers generally learn to think in terms of individual variation, and in terms of distinguishable capacities—a very different attitude from the traditional one heretofore cultivated."

The classification of pupils into three groups might be the work of an expert psychologist. The work of the psychologists in classifying men for service in the recent war might be extended to the schools with advantage. Provision should be made for re-examination and re-classification into higher groups as the pupil develops. The differences within such groups should not be so great as to discourage anyone; indeed there should be an incentive for all in the possibility of promotion to a more rapidly moving group. As conditions are, if we set the pace for the medium group both the other groups suffer, if we set it for either of the others matters are much worse. Mathematics probably suffers more than any other study from this situation.

Some improvements are being made, chiefly in the large centers and in junior high schools, but scarcely a beginning has been made.

In some localities the importance of teaching pupils how to study is recognized by the introduction of supervised study. One of the chief difficulties with supervised study lies in the fact that teachers themselves do not know and are not instructed how to use to advantage the extra time given a class for this purpose. Some teachers consider it merely an opportunity for the pupil to study under favorable conditions, where home conditions are not well suited for study. Others think of it as a time for the pupil to consult the teacher for help when he finds a problem which he cannot do, and the pupil is assisted with this difficulty. Yet others look upon it as a time when the school authorities wish the pupils to work, while the teacher has opportunity to do work of his own, read a story, etc. And others use it as extra time for teaching the subject. If these are the only uses made of the extra time, then supervised study is a complete failure. The mortality of the classes may be reduced, but this may mean nothing. The teacher merely feels that the pupil has done the best that could be expected of him, that he has spent sufficient time studying, that he would probably gain little from repeating the course, and that the supervised study period is intended to reduce failures, hence the pupil is passed on without due regard to scholarship. Such procedure involves the pupil in endless difficulty later unless the same policy is used throughout his student career. However, to tell a pupil he is educated when he is not educated is to give him false values of life, it is to encourage Bolshevism. The schools with such a policy would become a menace to, rather than a guardian of, our national interests.

In the proper use of a supervised study period the teacher must first of all know how to study, he must then teach the class how to study. The pupils should then apply the information and training given them to the lesson in hand. Some weak members of the class may need individual assistance, but this should be given in such manner as to teach the pupil how to do his work without help. The pupil must not feel dependent on the teacher while he studies. He should study and master the



lesson as he will later meet problems out of school which must be mastered. In the case of pupils who may have learned how to study; the teacher must not interfere with their preparation of the next lesson. Such pupils may be used to the advantage of the other members of the class. They may suggest effective methods for study. However, if the teacher knows how to teach, he will be constantly teaching how to study as he teaches the subject. In such case there will be no need for a supervised study period, except for backward pupils, and for those who have home conditions unsuited for study. After a lesson has been properly taught, and if the pupil has given his attention and interest to the work, he should feel that further work assigned on that topic is purely his own personal responsibility. Unless, then, supervised study periods are properly used we can get along better without them.

We are indebted to Mr. S. A. Douglass, Principal, Central High School, and Principal of the Summer Session of the Soldan High School, St. Louis, for the following information concerning an experiment which he made in the summer of 1916. We give a brief account of it here because it emphasizes the importance of teaching pupils how to study, and it contains valuable suggestions for those interested in helping to solve the problem.

A printed card was given each pupil. The front of the card contained blanks for marks on recitation, study, and standing, to be filled in by the pupil each week, and these were to be checked by the teacher, who made such suggestions as he thought necessary. The pupil was also to indicate place of study, time of study, and amount of study. The back of the card contained the following excellent suggestions on how to study:

*I. Conditions Favorable to Successful Study.*

1. Study away from interruptions as far as possible.
2. Have a definite study program and follow it faithfully. Give to each subject its just share of your study time.
3. Study your lesson as soon as possible after the assignment is made.
4. Concentrate your mind so that outside interests will not frequently disturb your study.
5. The use of the dictionary and reference books promotes good work.

*II. Attacking the Lesson*

1. Make sure that you clearly understand the subject of the lesson, or the particular problem to be solved.
2. Find the important facts in the new lesson and connect them with the facts previously learned.
3. Group the minor points of the lesson about the leading topics, thus making an outline of the work in hand.
4. Do not try to commit exact words until you understand the content of the sentence or paragraph. Mechanical memorizing is never advisable.
5. Make comparisons and contrasts when possible.
6. Review frequently. This greatly aids in the assimilation of knowledge. Reflective thinking is eminently worth while.
7. Make up your mind that you can learn. A strong will can accomplish wonders. Difficulties fade away for the person with an unflinching determination.

*III. Meaning of the Marks to be Used*

“E” means that you have grasped the subject; thought about it; made it your own; so that you can give it out again, with the stamp of your individual insight upon it.

“G” means that you have taken it in and can give it out again in the same form in which it came to you, but the words come from the book or the teacher and not from you.

“M” is much like “G,” only that your second-hand information is partial and fragmentary, rather than complete.

At the close of the session the pupils were questioned with the following results:

Out of 1887 responses:

1. 1,536 kept the record; 281 kept it partially; 56 neglected it entirely.
2. 1,456 kept it day by day throughout the term.
3. 1,046 thought the keeping of the record had helped them in some way; 444 were uncertain as to the results; 383 said it was no help.
4. 900 said they had carefully studied and followed the suggestions on the back of the card; 805 had done so partially; 142 had given the suggestions no attention.
5. 875 recommended some such plan for the regular school term; 685 opposed its use in this way; 271 were non-committal.

A few of the answers as to how they had been helped are:  
I am able to see whether the week has been successful or not.  
The suggestions helped me to grasp things more easily.  
It showed me what I was doing and made me work harder.  
It gave me a better idea of the teacher's view of my work.  
In order to put a good grade on my card I worked harder.  
I profited by the suggestions made by the teacher.  
It told me whether I was overestimating my results.  
It helped me to know how I stood in class.  
It gave a more comprehensive idea of work value.  
I could tell whether or not I had studied enough.  
It helped me to find out what to study most.  
It gave me confidence in myself.  
It reminded me of my past record, spurred me to study, and made me more careful in my study.  
I noticed the ratio between study and recitation marks.  
It gave me greater accuracy in the judgment of my work, and in valuing my work.

All the suggestions on the back of the card were recognized as helpful, but the following were emphasized most:

Study away from interruptions as far as possible.

Study your lessons as soon as possible after the assignment is made.

Review frequently. This greatly aids in the assimilation of knowledge. Reflective thinking is eminently worth while.

The teachers were also questioned as to the result of the experiment, with results as follows:

1. Practically all the pupils were perfectly willing to keep the record.

2. Most of the pupils showed care and fidelity in marking the card, some were indifferent and a few were doubtless dishonest.

3. There was only fair evidence that the pupils tried to carry out the suggestions on "How to Study."

4. There was considerable evidence that the pupils do not understand the meaning of marks in the best sense of the term.

5. In general the spirit manifested by the pupils in carrying out the details of the plan was good. Some were indifferent.

6. About half the pupils showed a commendable degree of

accuracy in estimating the worth of their work. The poor and indifferent pupils frequently overestimated the value of a recitation.

7. The very best pupils showed dissatisfaction with the plan, especially the marking part of it. The younger pupils were enthusiastic about the plan, while the older ones were not.

8. The teachers were unable to report any noticeable improvement in the pupils' method of study, although many of the pupils themselves reported that their method of study had improved.

9. The teachers generally recommended the follow-up plan (checking by the teacher) and some thought it absolutely necessary. Personal interviews generally brought results, but in some cases pupils continued to mark themselves too high.

10. The chief objection to the cards was that they unduly emphasized marks.

11. The card was thought of value to certain classes of pupils and of little value to others.

12. The best results seem to have been reaped by medium pupils.

13. The best and poorest pupils got little or no benefit; the former because they already had good methods, and needed no stimulus; the latter because of indifference and innate slowness.

14. The teachers thought the suggestions on the back of the card most valuable. The follow-up plan was ranked next in importance.

15. A small majority of the teachers thought it worth while.

16. It was not thought advisable to recommend the plan for the regular term without further trial and study.

17. It was recommended that pupils report the time spent in the preparation of each lesson rather than on the quality of their study.

18. It resulted in a better understanding between teacher and pupil, and a more systematic preparation of the daily work.

19. The suggestions on how to study should be enlarged and made more specific.

The purpose of the experiment seems to have been to determine what might be done towards improving the study habits of pupils. No very definite conclusions seem to have been

reached. The weight of responsibility seems to have fallen upon the pupil rather than upon the teacher. In case the pupil has bad habits of study, or no habits of importance connected with study, it is not probable that the pupil will have either understanding or initiative sufficient to enable him to form good habits. The teacher must be held responsible for teaching him how to study, if any real improvement is to be expected. Such an experiment is of value in emphasizing the importance of the matter, and in suggesting how to attack the problem to advantage. A summer session is too short and too hurried a time in which to apply the remedy effectively. We might almost as well expect the pupil to teach himself as to expect him to place correct values on his own efforts and achievements. It is worth while to know what he thinks, but only for the purpose of assisting him in the most advantageous way.

The fact that the best pupils saw no value in the cards is not necessarily an argument that they did not need instruction in methods of study. They may still fall far short of their possibilities. Many of those who have attained some measure of success in life realize that valuable time and effort might have been saved if they had been taught in the beginning how to study, and that some of the bad habits formed were later almost impossible to break.

Such experiments as this by Mr. Douglass, and all efforts to teach pupils how to study, will eventually result in the successful solution of the greatest problem that confronts us in our teaching at the present time.

Correct methods of study involve the proper use of text books. Every pupil should own the book he uses for a text. Not only this, but the book ought to be kept for future reference. Books are for the purpose of aiding the student to master the subject, and should be used in any manner that will help. They are not to be preserved to bring the highest possible price as soon as the course is ended. They should be underscored, have notes written on the margin, or interlined. In this way they can be made convenient for quick and ready reference, or in any case, they can be made to assist the owner.

It is for this reason that free texts are undesirable. Of course a borrowed book is better than no book but if pupils are unable

to buy texts, a way should be found by the community, by which they can own the books they use. Again free texts are undesirable for sanitary reasons. They are frequently passed from one pupil to another without any attempt to clean them, and in this condition they may be the carriers of disease. Yet again, some of the texts become decidedly shabby before they are discarded, and since a pupil has no choice but to take the shabby book if such is offered him, a strong dislike for the subject may originate in the objectionable book. While there are some strong arguments in favor of free text books, the writer believes the objections to them are much stronger; one of these is, the pupil cannot be taught some of the important points in the art of study if he does not own the book he uses.

The pupil should form the habit of noting the author, title, publisher, and date of publication of the books he reads. He should, if possible, learn something about the author. In this way he can learn to compare authorities and form independent judgments of his own regarding them. He should study the contents to obtain a general notion of the material covered in the book; and he must learn to use the index for purposes of reference. It is amazing how few pupils seem to know that there are such parts to the book, much less that these parts can be of any importance to them. If they have noticed these parts at all it has been with the idea that in some way they were necessary for every book, but that no one thinks of using them.

Text books ought to be up to date; not only that, but they ought to be selected with care as the best available of the modern books. No text in plane geometry is satisfactory which does not provide constructions and informal proofs at the beginning of the course. It should postulate theorems that appear to the average student as self-evident. It should provide numerous applied problems, connecting the subject with other fields; and some reference should be made to the trigonometric functions, etc. No text in algebra is satisfactory which does not relate the work to arithmetic; which does not use the graph; which does not emphasize the function, and the equation; and which does not give wide opportunity for the development of the reasoning powers in thought problems; as well as exercises for the mastery

of technique. We need a better type of book than we possess for mathematics. So far they are dominated by the interests of the subject. We need texts constructed according to the interests of the pupil, since he is the most important factor in the consideration. We read books arranged and graded according to the ability of the pupil to learn the subject. Professor F. M. McMurray, in "How to Study," says that text books should contain abundant detail:

"Without plenty of detail the facts have to be run together, or listed, merely as so many things that are true; they then form only a skeleton, with all the repulsiveness of a skeleton."

It is claimed by some persons that a teacher should be able to teach without a text, or to make his own text. No doubt the teacher should be a thorough master in his subject, but he may not be able to write a text book. The making of text books requires special abilities and few teachers possess them. It is no reflection on the ability of a teacher if he cannot write a text book. Moreover texts should be the outcome of wide experience in the teaching of the subject, it is probable that they can be written better by a group of teachers than by a single individual. It is a waste of time and energy to teach mathematics without a text, and few teachers try it. A poor text is better than none. Professor L. V. Koos has shown as the result of an investigation including 416 schools in 15 states of the North Central Association of Colleges and Secondary Schools, that:

"Text books dominate content and organization of courses in mathematics (in high schools)."

Commenting on the same topic, Rugg and Clark, in "*Reconstruction of Ninth-Grade Mathematics*," say:

"It seems evident that *text books almost always completely determine the specific subject-matter that is taught to students in the course.*"

The same writers divide high school teachers of mathematics into three groups:

1. The typical mathematics teacher, with no specific training in mathematics beyond a one or two years' course in college mathematics, and no training in teaching mathematics, and only one, two, or three years' experience.

2. A group of teachers in larger cities, who have longer experience, fairly adequate college training in mathematics, but almost no training in the teaching of mathematics. These change the order of subjects and supplement the material of the text.

3. A small group, well trained in content and in professional courses. Many of these are either independently or in connection with college men, textbook writers.

We have seen that the advantages from the study of mathematics cannot be properly realized unless pupils are taught how to study, that this matter is very greatly neglected at present, and that many elements enter into the successful teaching of the art of study as it relates to mathematics. Some attention is being paid to the matter but it still remains our most serious problem.